

# **SWEPT SINE VIBRATION TEST CONSERVATISM**

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The excessive overtest associated with the swept sine vibration test method was measured quantitatively using the index of conservatism and the associated overtest factor for a dynamic mass model of a typical spacecraft component. The response to a fixed amplitude sine sweep test was compared to the flight transient vibration environment for sweep rates of 2, 4 and 6 octaves per minute and 300 Hz. per minute. A response limited test was also conducted at 6 octaves per minute. The conservatism was measured using several characterizations, namely: number of peaks exceeding, ranked peaks, shock response spectrum, shock intensity, three dimensional shock response spectrum and ranked peaks. Overtest factors exceeding an order of magnitude were measured for the test response with the number of peaks exceeding and three dimensional shock response spectrum.

## **INTRODUCTION**

The swept sine vibration test is used extensively to validate spacecraft components and structures for launch loads. It is simple to control and execute but suffers from the drawback of producing considerable overtest. This overtest has been alluded to in the past but never quantified. For instance in reference [1], the maximum peak responses produced by a sine sweep and other transient test methods were compared subjectively. Experiments were therefore conducted to quantify the overtest associated with using the swept sine test. These experiments consisted of applying a swept sine wave to a dynamic mass model of a radioisotope thermoelectric generator (RTG), shown schematically on a shaker table in Figure 1. A single launch transient response, Figure 2, at the RTG free end, was used as a reference for quantifying the overtest produced. The corresponding transient at the RTG base is shown in Figure 3.

Several sine sweep tests were run consisting of a 1.5 g(0-pk) fixed amplitude shaker table motion swept once up from 10 to 100 Hz, at different sweep rates. A 2 octave per minute sweep rate is a typical test requirement for assemblies such as the RTG's in the Cassini spacecraft. The amplitude was chosen to be approximately 1.5 times the 95 th. percentile amplitude of the calculated modal accelerations predicted for several launch events, as shown in Figure 4, for the RTG base. This is a typical protoflight test requirement for a sine test amplitude for the environmental testing of spacecraft subsystems and assemblies [2]. The subject RTG base transient event of Figure 2 is only one of the events shown in Figure 4, It should be remarked that the modal accelerations at the RTG base were obtained from a coupled loads analysis with a low damping ratio of about 0.05 and a crude stick model of the complex RTG component. The realistic launch transient free end response of Figure 2, was obtained from the above base

acceleration and the measured RTG frequency response function which included a damping ratio of about 0.13. This provided a realistic response which could be duplicated by applying the base transient to the RTG in a test.

If modal response information, is not available a response limited sine test would be run, with a shaker table input amplitude similar to the maximum flight amplitude of the RTG base. A response limited sine test was therefore run to simulate this case with a shaker table amplitude of 2.0 g (see Figure 3) at a sweep rate of 6 octaves per minute. The test response limit was obtained from the maximum peak (4.7 g) of the corresponding calculated RTG free end response of Figure 2.

### CONSERVATISM INDICES

The overttest occurring in a test may be quantified using the index of conservatism (IOC), [3]:

$$IOC = \frac{\bar{C}_T - \bar{C}_F}{\sqrt{\sigma_T^2 + \sigma_F^2}} = \frac{M}{\sigma_M} \quad (3)$$

where  $\bar{M}$  is the mean margin of conservatism and  $\bar{C}_T$  and  $\bar{C}_F$  are the mean transient characterization values for the test (T) and flight (F) environments, and  $\sigma_M$ ,  $\sigma_T$  and  $\sigma_F$  are the corresponding standard deviations. The suffix M refers to the combined mean of the flight and test characterizations. In practice several tests would be run and characterized and an averaged characterization generated together with the above statistics. The IOC measures the probability of achieving an overttest given the statistics of the test and flight environment characterizations. For instance IOC values of zero, one and two correspond to 50, 84.1 and 97.9 percent probabilities of an overttest occurring.

The IOC quantifies the probability of an overttest but not the amount of overttest. This quantitative information is provided by the overttest factor (OTF) [3]:

$$OTF = \frac{C_T}{C_{T,I}} \quad (4)$$

where  $C_{T,I}$  is the mean characterization of the test data which produces the desired IOC value of 1. The OTF defines how many times greater the actual mean test characterization,  $C_T$  is than the mean test characterization,  $C_{T,I}$ , having an index of conservatism of I. If one assumes a constant ratio between the test and flight environments then:

$$R = \frac{C_T}{C_F} \quad (5)$$

and:

$$R_I = \frac{\overline{C_{T,I}}}{\overline{C_F}} \quad (6)$$

The IOC is then expressed as:

$$I = \frac{R_I \overline{C_F} - \overline{C_F}}{\sqrt{\frac{2}{k_F^2} + \frac{2}{k_T^2}}} = \frac{(R_I - 1)}{\sqrt{k_F^2 + R_I^2 k_T^2}} \quad (7)$$

where  $k_F$  and  $k_T$  are the coefficients of variation for the flight and test environments, respectively. Equation (7) can be solved for  $R_I$ , and the OTF is found using equations (6) and (4). A value of 1.0 was chosen for the OTF calculations herein, since this represents a reasonable degree of conservatism that would be sought in a component level vibration test. The OTF was used exclusively to quantify the overtest of the sine sweep test method.

## CHARACTERIZATIONS

The conservatism measurements were made using several characterizations of the test response. The traditional shock response spectrum (SRS), three dimensional SRS (3 D-SRS)[4] and the number of peaks exceeding a given response level were used. A further characterization used was that of peak ranking (PKA), originally developed for shock measurements [3]. This ranks the positive and negative peaks in the test response according to magnitude. Shock intensity (SI) was also used as an adjunct to the SRS, to provide an estimate of the shock energy imparted to the test article.

## SINE SWEEP TEST RESPONSES

The sine sweep test acceleration responses were digitally recorded directly to computer files at a sampling rate of 512 Hz., after passing through a 200 Hz low pass filter for antialiasing control. This provided a reasonable compromise between the need to obtain frequency resolution up to 100 Hz. and the need for reasonable peak descriptions of the data, and the available computer analysis software limitations. The digitized test data was bandpass filtered between 10 and 100 Hz before the characterizations were calculated. The sine sweep tests were run with exponential sweep rates of 2, 4 and 6 octaves per minute and a linear sweep rate of 300 Hz. per minute. The corresponding responses are shown together in Figure 5, where the excessive response durations are evident compared to the subject flight event's duration of 1 second.

The maximum peak responses recorded with the different test sweep rates and the flight event are compared in Figure 6. The term RL refers to the response limited swept sine test. There is little difference in maximum response between the test and flight responses. This behavior is expected here because the known high damping of the RTG does not produce an excessive free end

response as would be the case for a lightly damped component, The response limiting notch produced by the shaker control system is therefore small and the resulting response not significantly different from the unlimited sine sweep tests. The sine sweep test responses are now discussed relative to the response characterizations,

### NUMBER OF PEAKS EXCEEDING

A traditional display for the response to a sine sweep test is the 'number of peaks exceeding' characterization, Here the number of peaks exceeding a given peak amplitude are plotted against those peak amplitudes. Figure 7 shows this for the sine sweep tests and flight event. The number of peaks exceeding the lower amplitude peaks are reduced with increasing sweep rate. The response limited sine test offers a small advantage over the similar unlimited test at the same sweep rate. This advantage is present despite the 33% larger amplitude of the sine sweep input. The sine sweep tests provide orders of magnitude overttest for all the test response peaks. The degree of overttest is shown in the OTF plot of Figure 8. Even the faster sweep rates overttest by about fifty times for the medium to higher level response peaks.

### PKA

The peak ranking (PKA) for the sine test responses, for positive and negative peaks combined, are shown in Figure 9 against that of the flight response. The peak rank scale stops at 80 since the flight time history data has fewer than 80 peaks. All sweep rates provide considerable overttest for the smaller amplitude peaks, The response limited sine. sweep test provides minimally reduced overttest at the higher magnitude peaks (ranks 1-20) than the other sine sweeps. This again is probably due to the high damping of the RTG producing comparable responses for the different sine sweep rates. The corresponding OTF plot is shown in Figure 10 for an IOC of 1.0, which represents a reasonable degree of conservatism that would be sought in a practical test [5]. If the OTF is averaged over the abscissa range then simple averages maybe used to compare the PKA overttest as in Figure 11. Thus all of the sine sweep tests show excessive average overttest ( $> 3$ ) for the smaller amplitude peaks below the 20th rank, Sweep rate or response control appears to have little effect on the average overttest for the PKA characterization over the narrow frequency range studied, for the high test article damping.

### SRS

The SRS characterizations for the test and flight responses are shown in Figure 12. Little difference shows between the various sine sweep tests, except around the resonant frequency of the RTG between 40 and 50 Hz., where the response limiting test response SRS is closer to that of the flight response SRS than the other sine sweep tests. This demonstrates the main advantage of the response limited test method in reducing the response amplitude around structural resonances. This advantage is more evident in the OTF plot of Figure 13, for an IOC of unity, where the OTF is significantly closer to unity around the structural resonance of the test article. Taking simple averages for the OTF over the frequency range of 10-100 Hz produces the

comparison chart of Figure 14. Thus all the sine sweep rates used provide substantial **overtest** over the frequency range shown, as evidenced by the average OTF's around the value of 3.0.

## S1

The shock intensity (S1) is an adjunct to the SRS and is calculated from the relationship:

$$SI = \int_0^f SRS ( \zeta, f_n ) df_n \quad (8)$$

Here, SRS denotes the shock response curve. This relationship follows from the argument of Housner [6], that the area under the shock response curve is a measure of the intensity of the motion in the sense that it expresses the average (of the maximum) response of the structure over the range of natural frequencies. Strictly speaking the S1 value should be divided by the frequency range to obtain a true mathematical average. The S1 values are displayed graphically in Figure 15, where the sine sweep tests are seen to all grossly overtest in terms of the S1.

## 3D-SRS

The 3D-SRS characterization clearly demonstrates the overtest associated with the swept sine test. Here the SRS is computed at several natural frequencies and then the number of peaks exceeding prescribed response levels calculated [4]. A three dimensional graph is then composed wherein the surface enclosing the points is called the 3D-SRS. Figure 16 is the 3D-SRS for the flight transient, where shading represents different levels of number of peaks exceeding. There is a pronounced promontory coming towards the reader along the 50 Hz natural frequency axis. Another way of viewing this is to eliminate the three dimensional surface and only show the number of peaks exceeding at each natural frequency and response as in Figure 17. This clearly shows the large response in terms of number of peaks exceeding at the 50 Hz natural frequency. The corresponding swept sine test response at 2 octaves per minute is in Figure 18. The response is clearly excessive even away from the resonance of the test article. This can be made clearer by using two dimensional graphs for each natural frequency as in Figure 19 for a below-resonance condition. Figures 20 and 21 show the response for the near-resonance and above-resonance conditions. Figures 22 through 24 are the corresponding OTF plots for an IOC of 1.0, which clearly show the excessive overtest of the sine sweep test method. Even at the lower response levels, the overtest factor is at least 10 and exceeds 1000 for the above-resonance condition of Figure 24. The sine sweep test therefore provides orders of magnitude overtest relative to the flight environment, for this characterization.

## CONCLUSIONS

The conservatism of the sine sweep test has been quantified relative to the flight response using the overtest factor (OTF), in terms of the PKA, SRS, 3D-SRS and number of peaks exceeding characterizations. - The sine sweep test methods have been shown to excessively overtest for all of the above characterizations for the representative sweep rates tested,

The high test article damping prevented a clear demonstration of the effectiveness of the response

limited sine sweep test in most of the characterizations used. However, the utility of this method in reducing the overttest around a structural resonance, was demonstrated by the frequency dependent shock response characterization,

The sine test sweep rate does not appear to significantly affect overttest in either the time domain peak ranking (PKA) or the frequency domain shock response spectrum (SRS) characterizations. Increasing the sweep rate reduces the number of peaks exceeding the higher peak response amplitude in linear proportion to the sweep rate increase.

#### ACKNOWLEDGEMENTS

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2. "Flight Equipment Classifications and Product Assurance Requirements". Jet Propulsion Laboratory, California Institute of Technology, JPLD-1489, Rev. B, January 1990.
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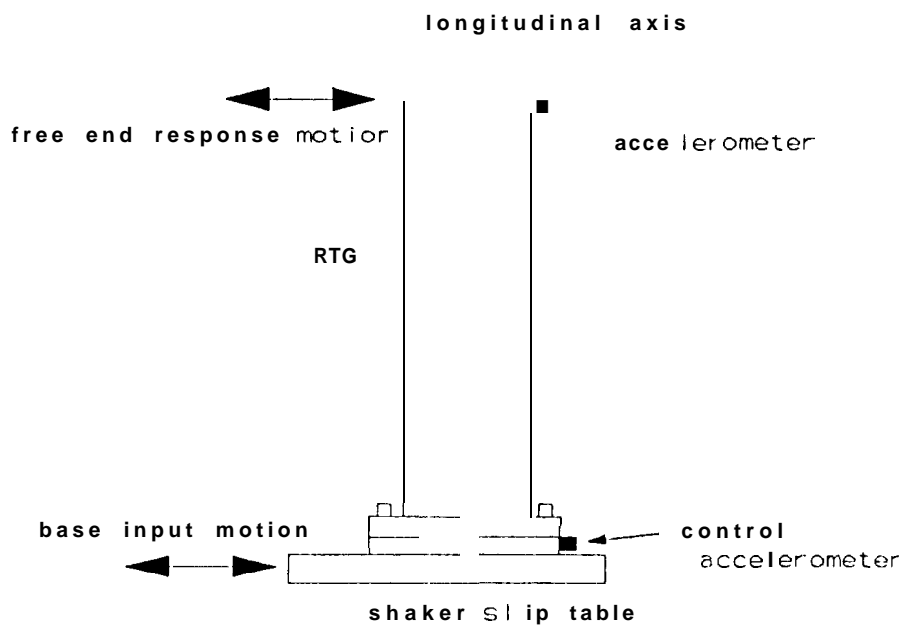


Figure 1. RTG on Shaker Slip Table

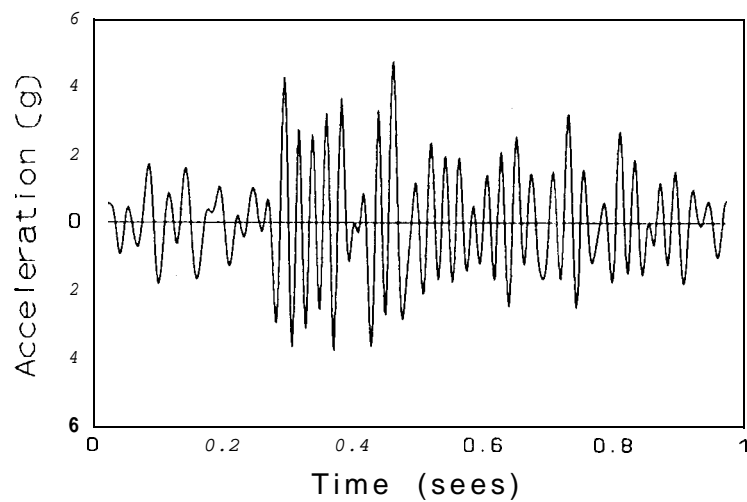


Figure 2. RTG Flight End Response

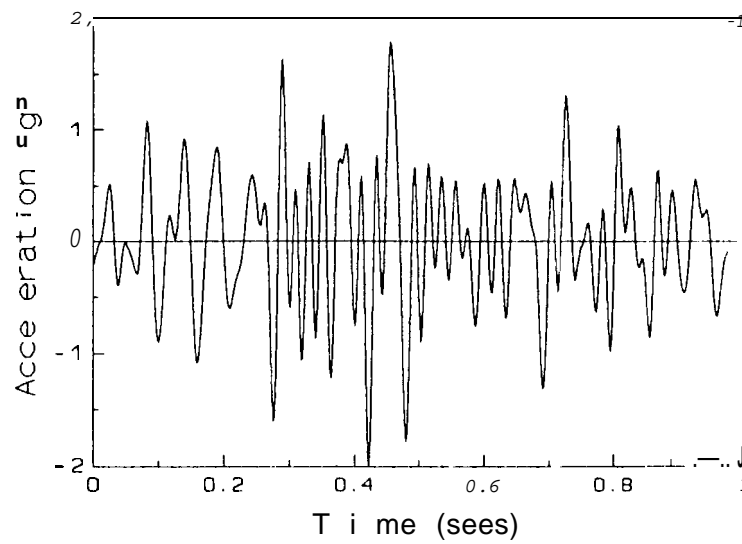


Figure 3. RTG Flight Base Input

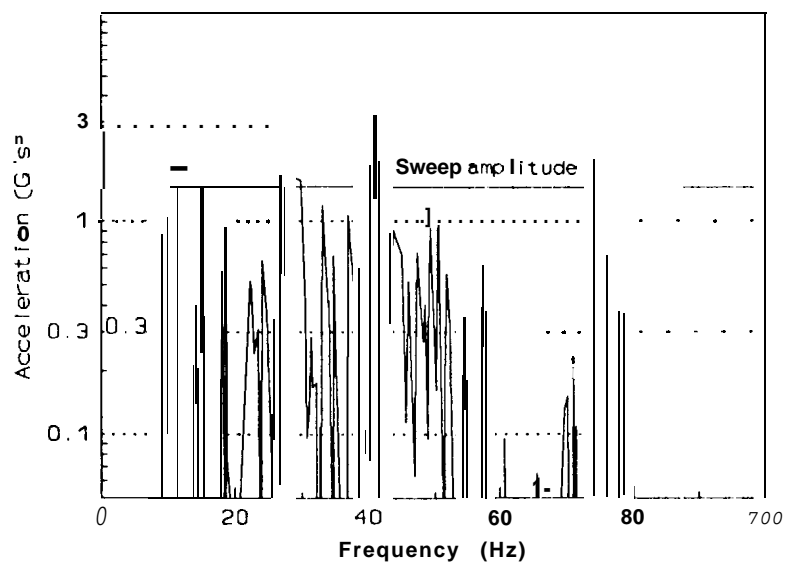


Figure 4. Swept Sine Test / Flight Levels



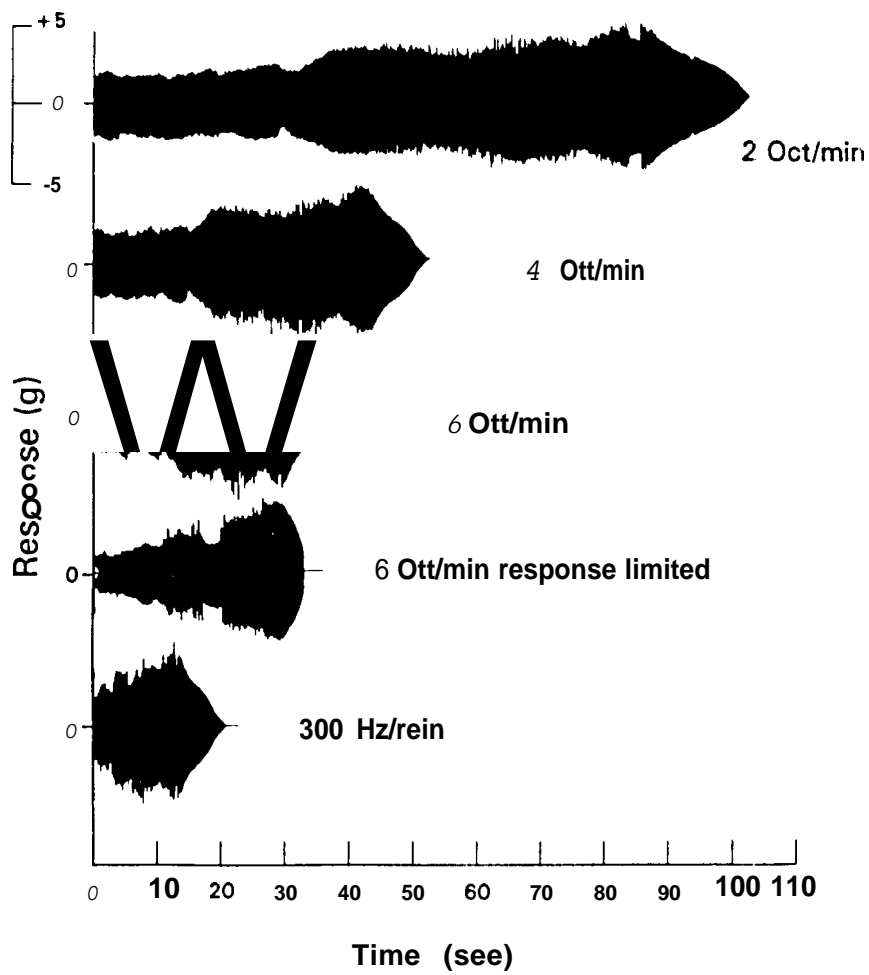


Figure 5. Swept Sine Test Responses

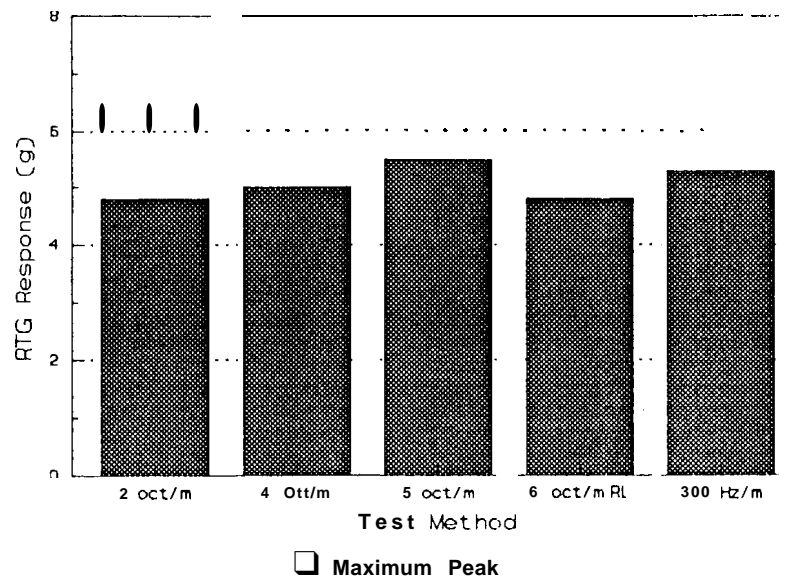


Figure 6. Maximum Peak Responses

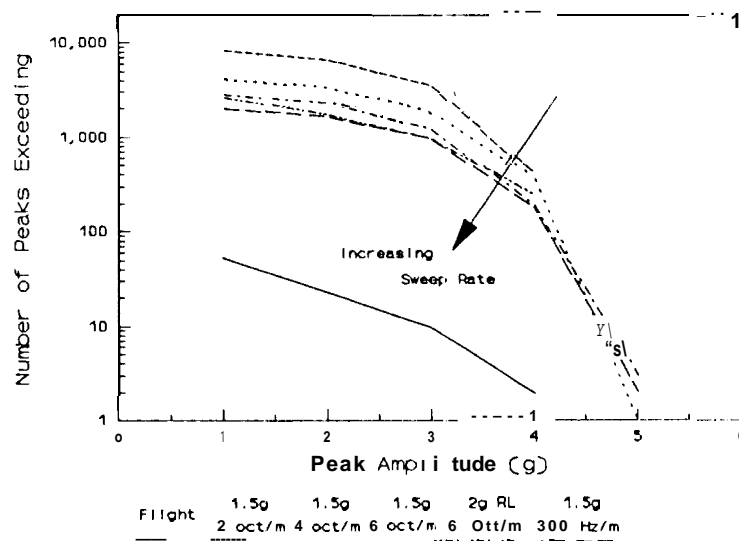


Figure 7. RTG Response - Peaks Exceeding

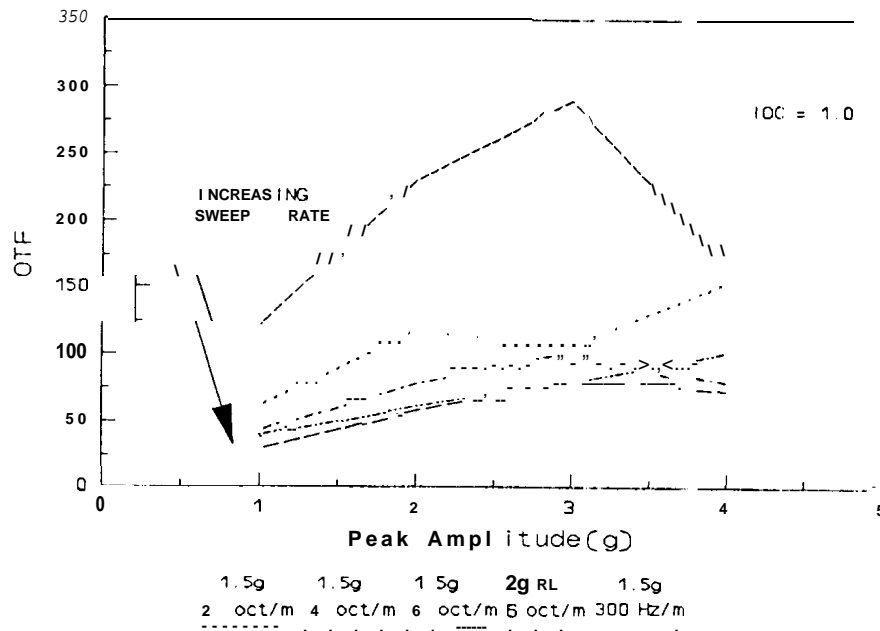


Figure 8. OTF -Peaks Exceeding

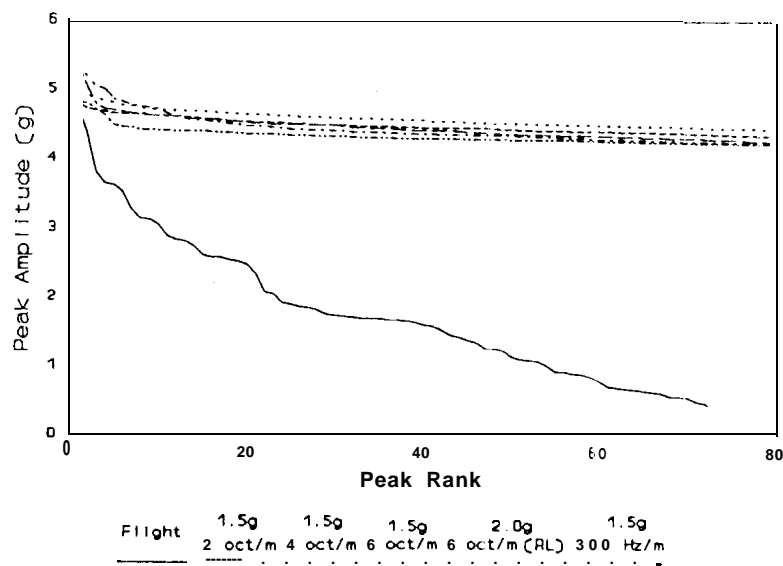


Figure 9. RTG Response PKA (+/-)

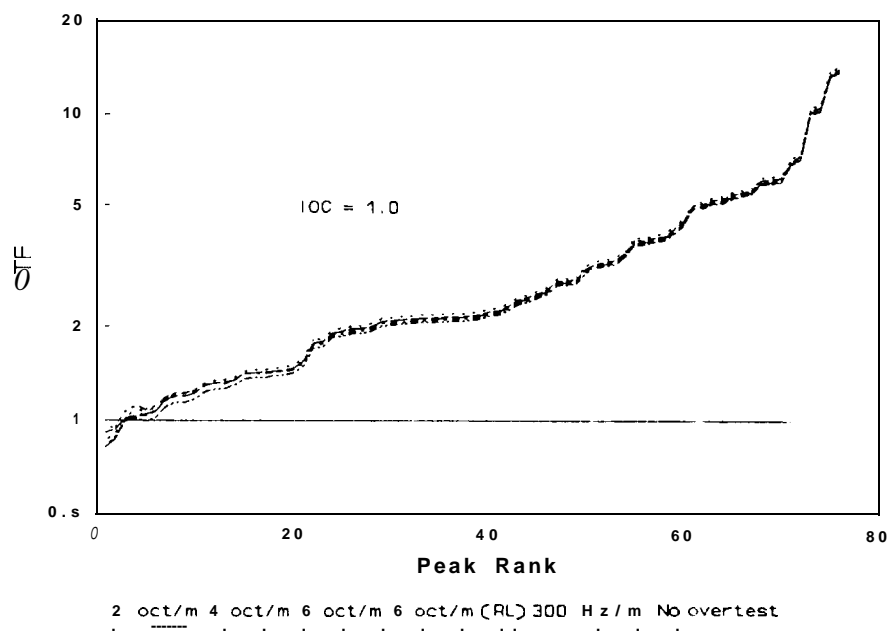


Figure 10. OTF for PKA(+/-)

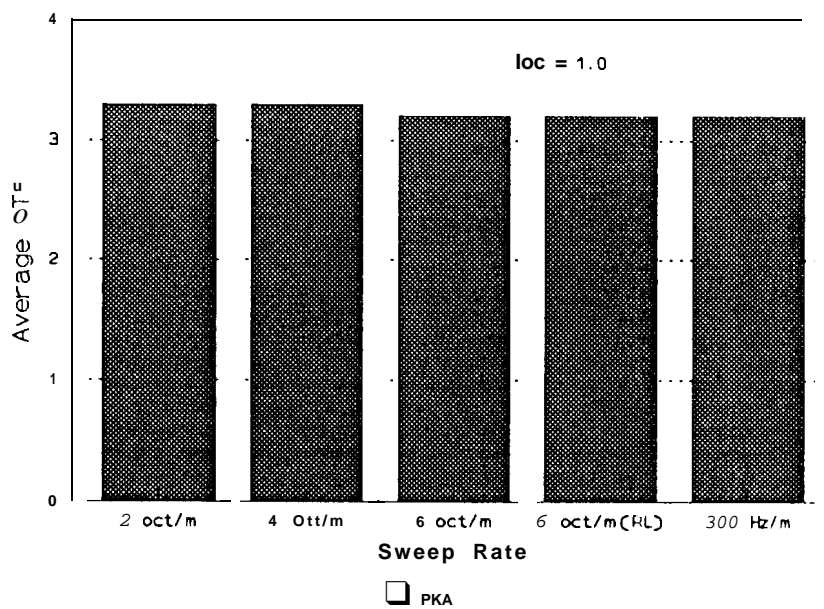


Figure 11 -Average OTF for PKA(+/-)

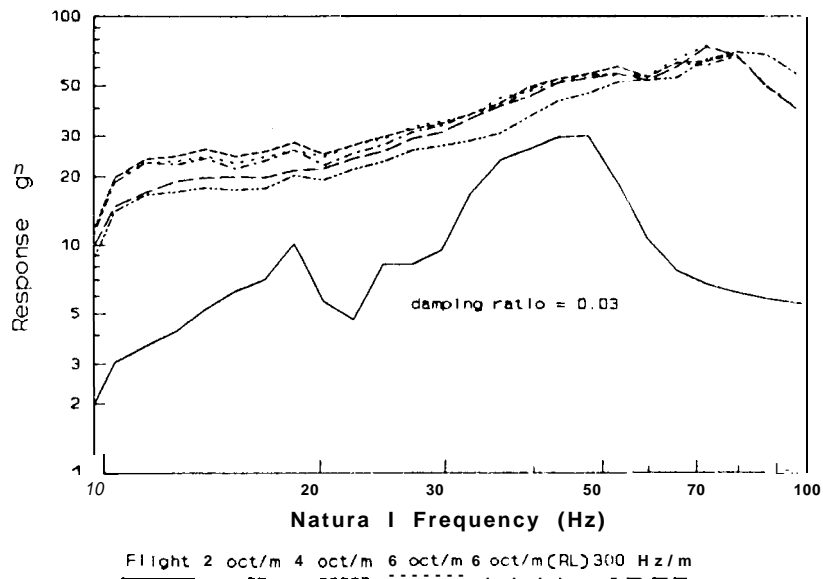


Figure 12. SRS of RTG Response

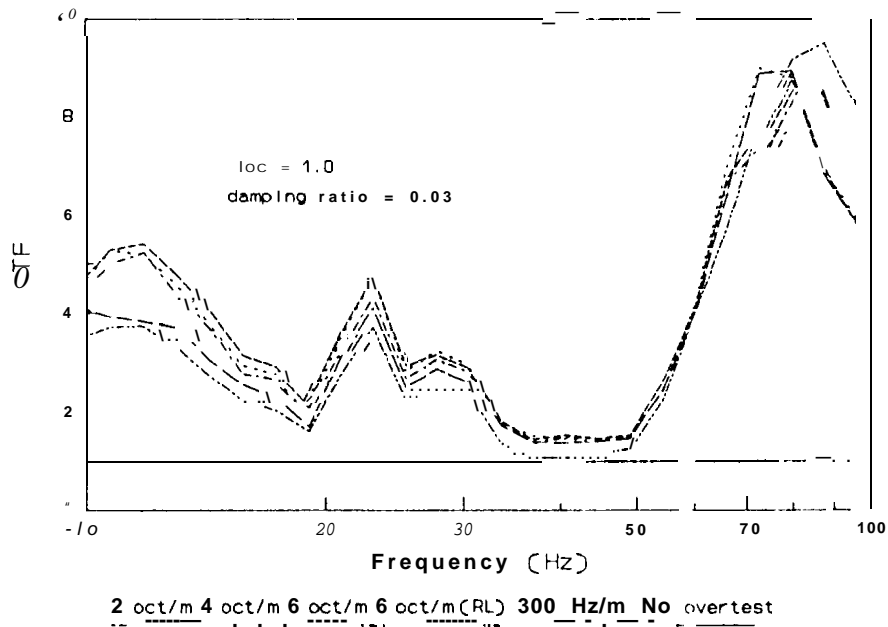


Figure 13. OTF for SRS of RTG Response

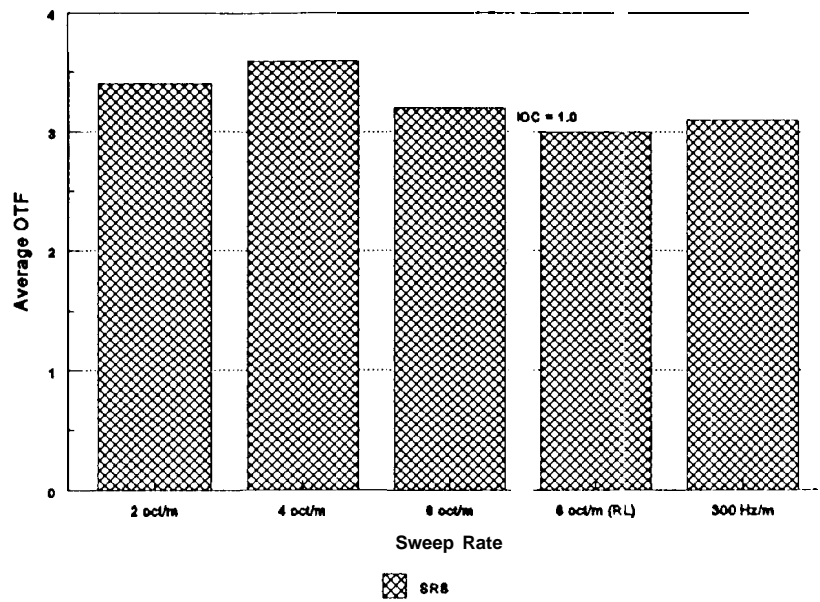


Figure 14. Average OTF for SRS of RTG Responses

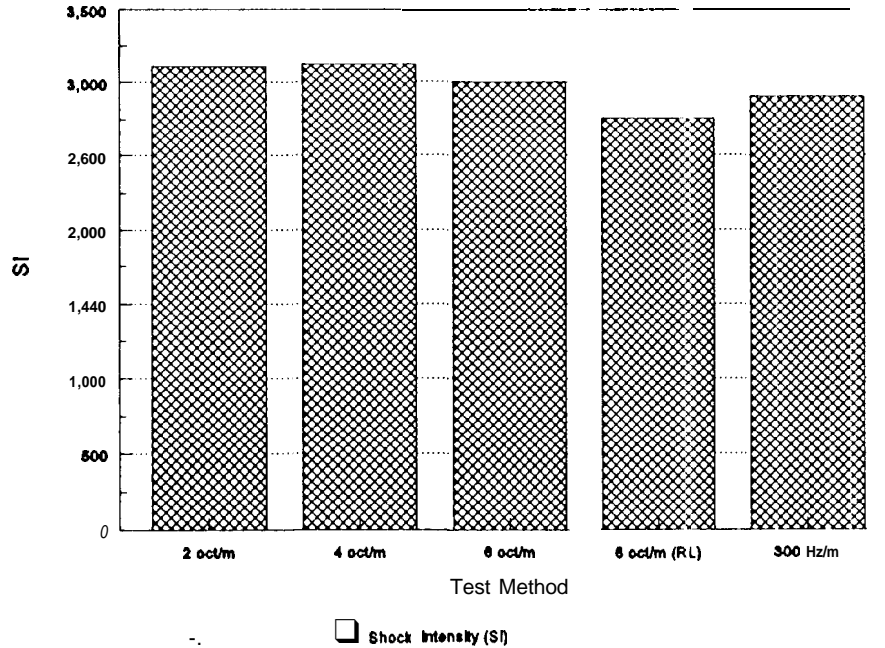
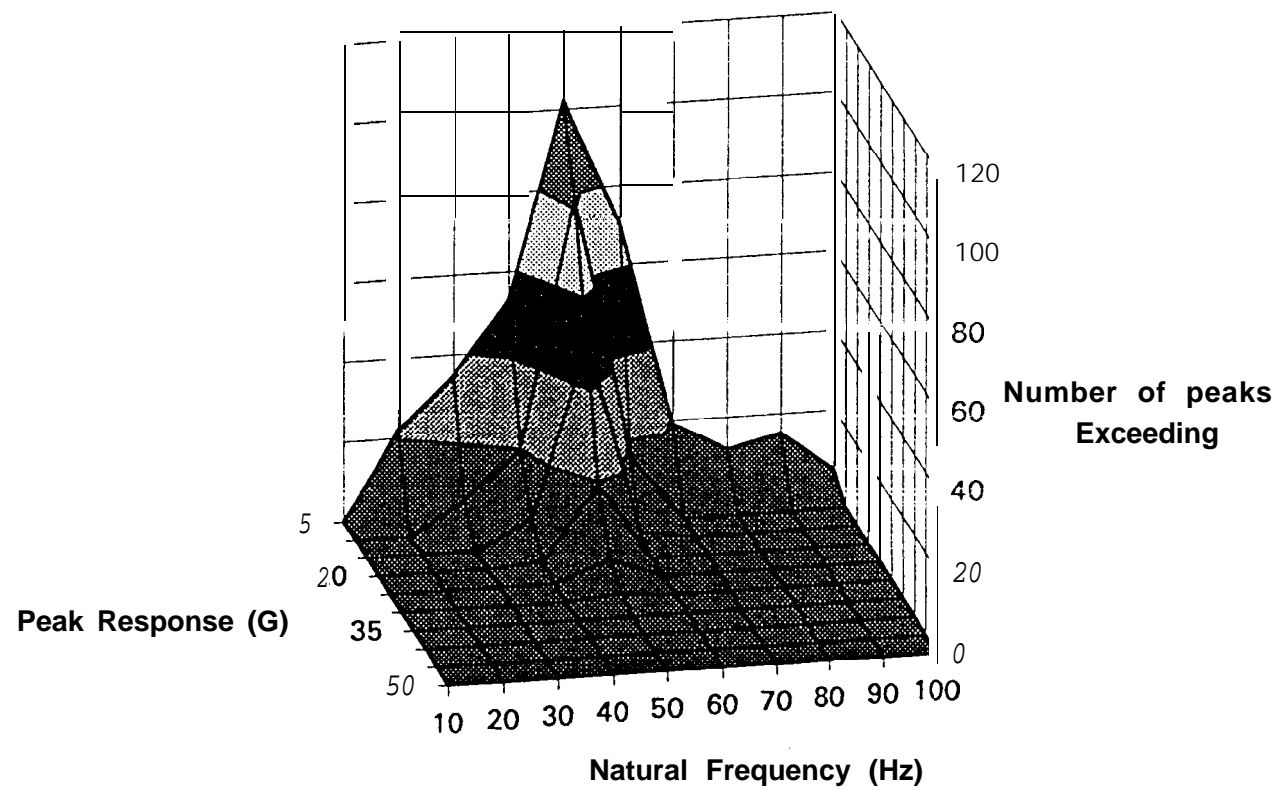


Figure 15. Shock Intensity (S1)



**Figure 16. 3D-SRS Flight Transient**

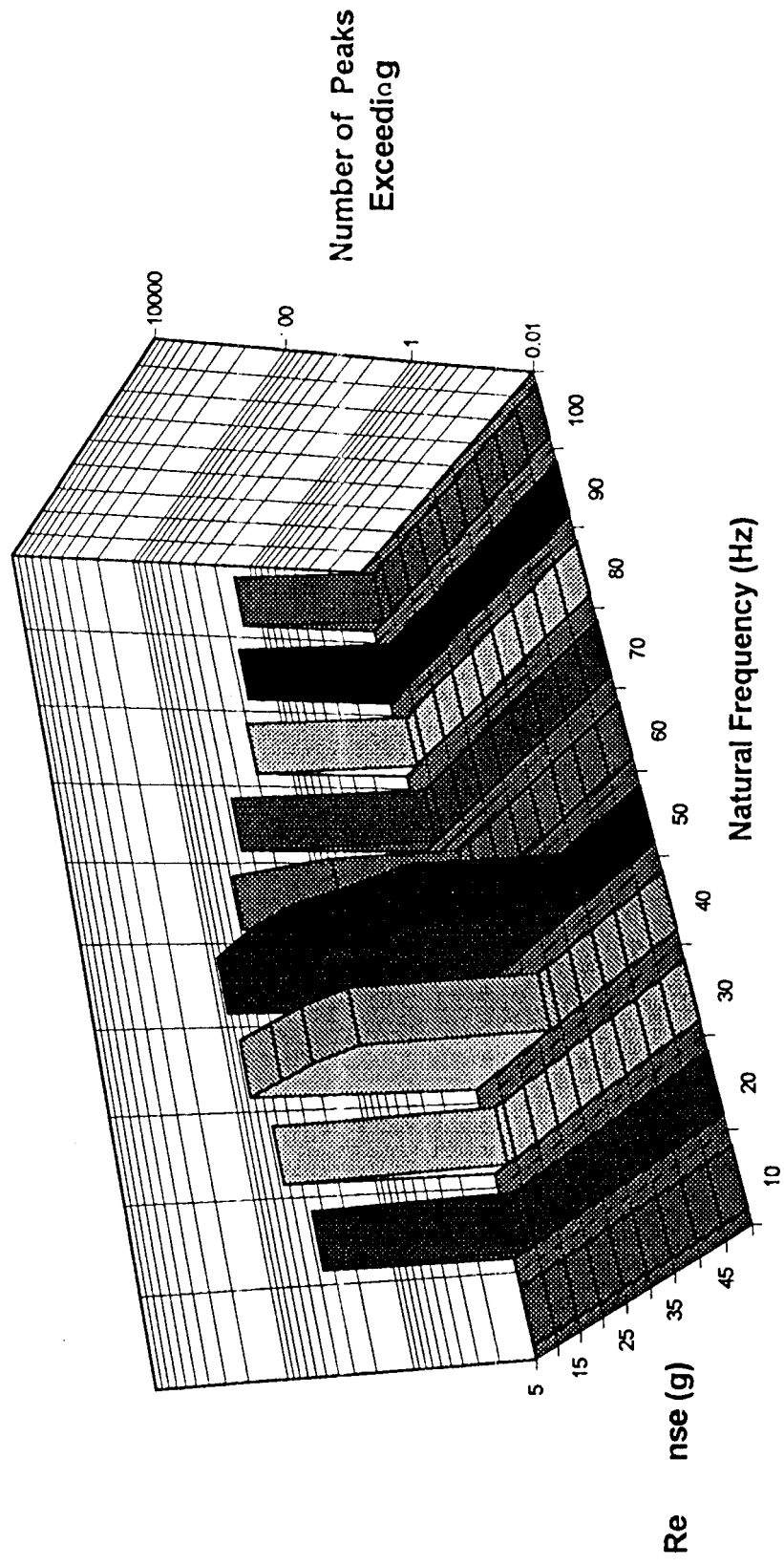


Figure 17 3D-SRS Components - Flight Transient



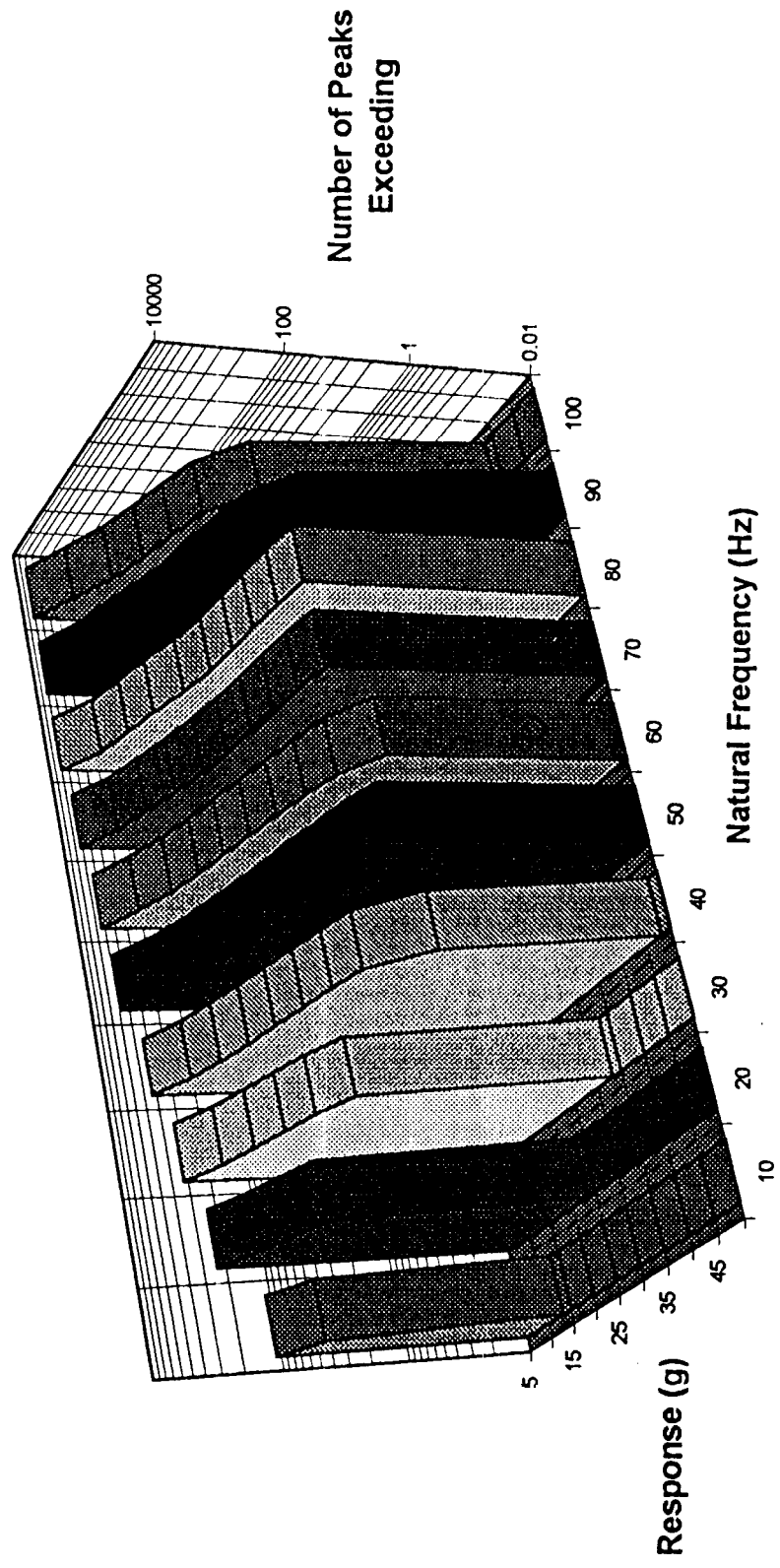


Figure 18. 3-D SRS Components - 2 oct/m Swept Sine Test

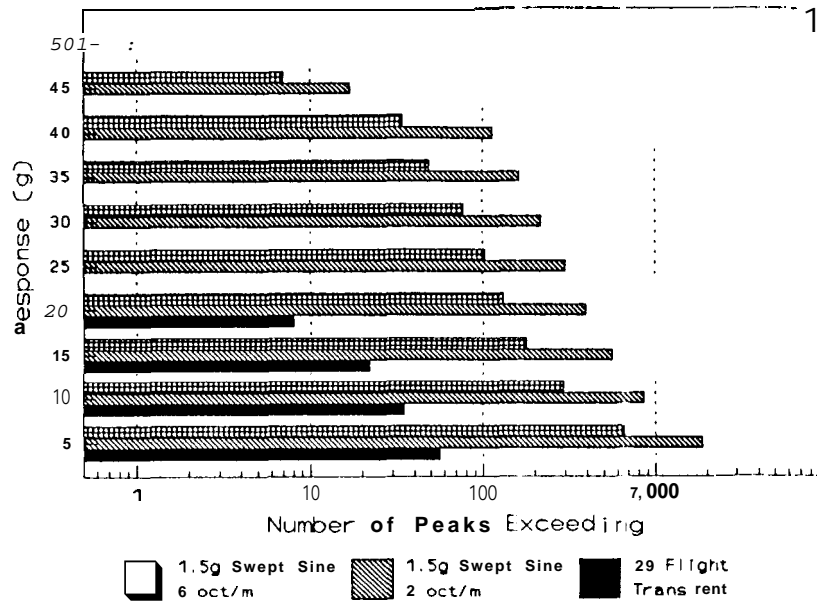


Figure 19. 3D-SRS at 40 Hz - Below Resonance

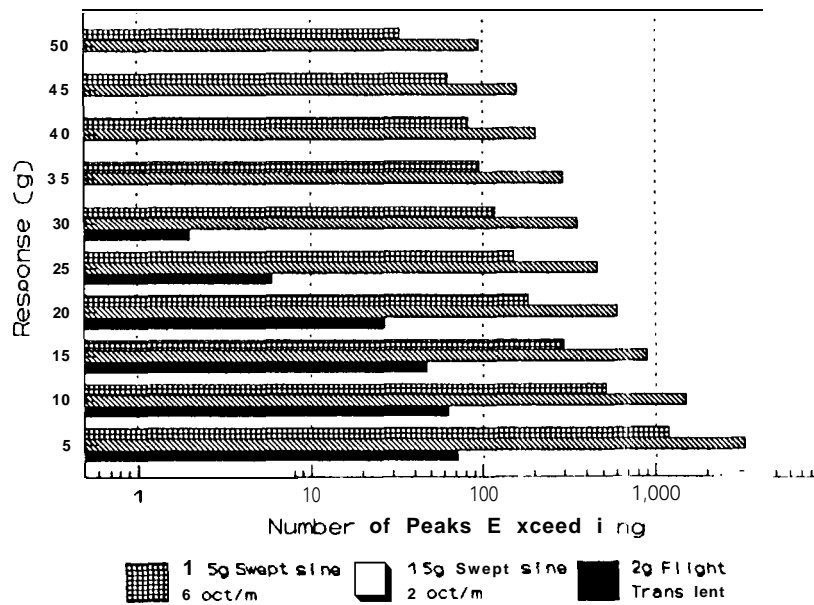


Figure 20. 3D-SRS at 50 Hz - Near resonance

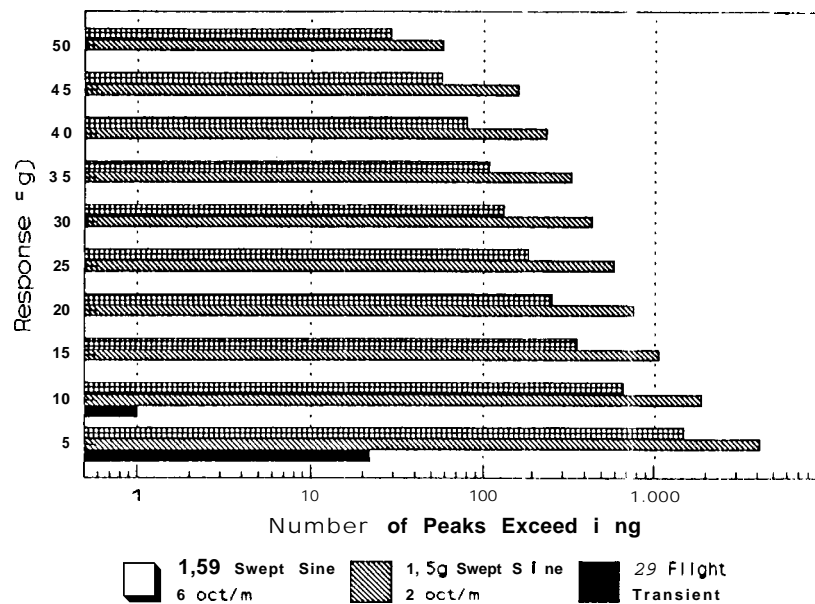


Figure 21. 3D-SRS at 60 Hz - Above Resonance

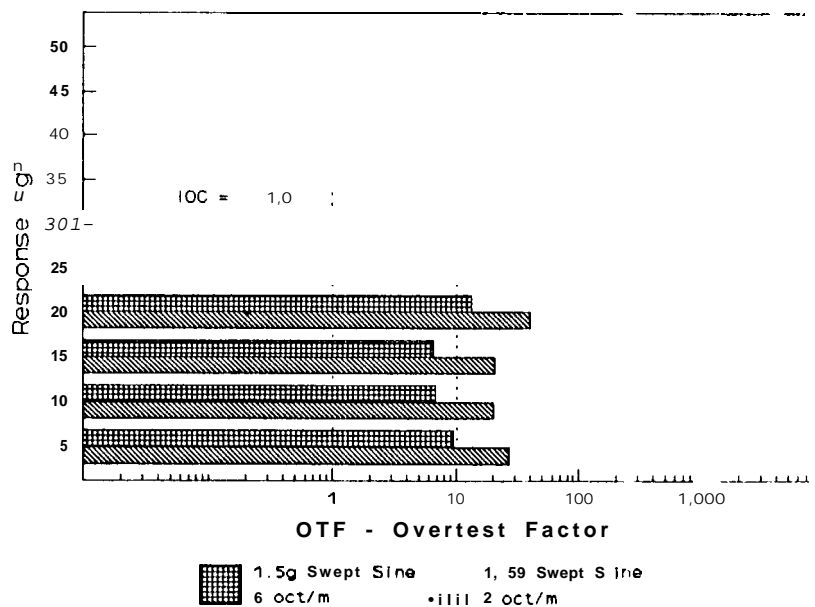


Figure 22. 3D-SRS OTF at 40 Hz - Below Resonance

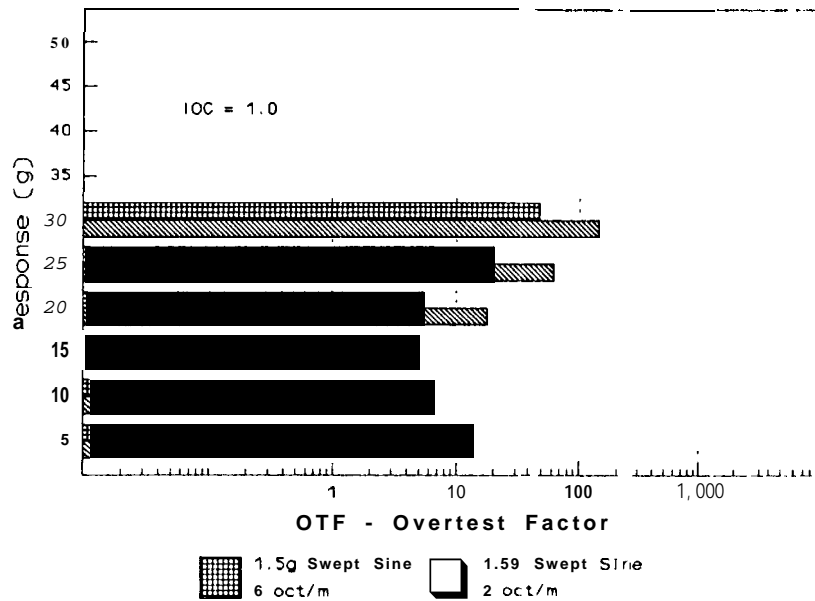


Figure 23. 3D-SRS OTF at 50 Hz - Near Resonance

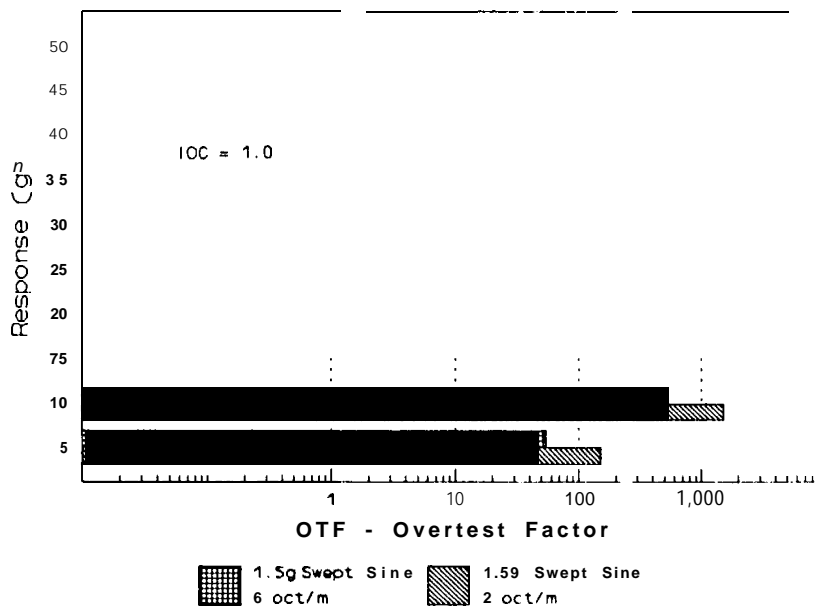


Figure 24. 3D-SRS OTF at 60 Hz - Above Resonance